Polymer model with annealed dilution on the square lattice: A transfer-matrix study

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We study a lattice model for equilibrium polymerization with annealed dilution. The model considered is an Ising lattice-gas monomer-solvent system where the polymers are represented by mutually and self-avoiding walks constrained to pass through sites occupied by monomers. Numerical results on the square lattice using transfer-matrix techniques and finite-size scaling are reported. The values obtained for the tricitical exponent v_t are in agreement with the tricritical Ising exponent (8/11) for high values of the monomer fugacity, but the accuracy we obtained does not rule out the possibility of v_t being equal to the critical self-avoiding-walk (SAW) value 3/4. Our results indicate that a crossover occurs in the tricritical behavior of the model. This crossover corroborates the equivalence between a particular limit of the diluted model and the self-attracting polymer system proposed recently.

I. INTRODUCTION

The thermodynamic properties of equilibrium polymerization in a solvent are often studied through models where the polymeric chains are modeled as self- and mutually avoiding walks on a lattice [1]. In a good solvent, the excluded-volume interactions between monomers incorporated into polymeric chains dominate, so that the exponent v (which defines how the root-mean-square radius of the chain scales with respect to the number of monomers in the chain) is equal to the self-avoiding walk (SAW) value 3/4 for models on two-dimensional lattices. On the other hand, when the monomers are in a poor solvent, a collapse of the polymer chains may occur, so that the polymers will be in a more compact configuration, and the exponent v changes to the lower random-walk value 1/2.

There are (at least) two different lattice models for studying the collapse transition of polymers. The first one is the so-called self-attracting self-avoiding-walk model (SASAW) [1,2], and the second one is the SAW model in the presence of annealed vacancies [3]. In the SASAW model, attractive interactions between monomers in nearest-neighbor sites on the lattice which are not consecutive sites of the same chain are included to simulate the interactions between the polymer and the solvent molecules. These interactions, identified with the short-range van der Waals forces, become important as the temperature is lowered, and a collapse transition occurs at the so-called Θ temperature [4]. This transition is identified as a tricritical point [1,2], but recently some doubts were presented in the literature related to the multicritical nature of the Θ point for models on two-dimensional lattices [5].

The model of equilibrium polymerization with annealed vacancies was proposed some time ago by Wheeler and Pfeuty [3], based on a chemical equilibrium model proposed earlier for the study of the polymerization of sulfur in organic solvents [6]. In this model the lattice sites may be occupied by monomer or by solvent molecules, the monomers may polymerize forming chains, and the interactions between molecules are modeled as an Ising lattice gas with first-neighbor interactions. Thus, the model is equivalent to the SAW problem on the lattice with correlated annealed vacancies. Mean-field calculations [3], Bethe lattice calculations [7], and some preliminary results on the square lattice [8] show that a collapse transition appears as a tricritical line in the temperature-lattice gas fugacity plane.

The diluted model has been recently related to the model of interacting polymers or SASAW [9–11]. In these references, it is argued that the diluted-polymer model may reproduce the self-attracting polymer system in a certain limit. The equivalence between both models in this particular limit was obtained analytically on the Bethe lattice [11], and some preliminary results for the model on the square lattice using finite-size scaling were published recently [8]. The equivalence between both models has been put in doubt by the argument that the tricritical behavior of the diluted-polymer system should be determined by the embedding Ising lattice-gas system, which has different tricritical exponents than the Θ point of the SASAW model [12,13]. This point was also discussed by Seno, Stella, and Vanderzande [14], who performed simulations for the diluted-polymer model which resulted in pure SAW critical behavior simulations for the diluted-polymer model which results in pure SAW critical behavior at the critical Ising point on the square lattice, e.g., $v_t = 3/4$, excluding the value $v_t = 8/11$, presented as the exact value in Ref. [13].

In the present work two different transfer-matrix calculations, using the phenomenological renormalization

group (PRG) and the first-order phase-transition method (FOM), are used to calculate the tricritical behavior of the diluted-polymer system and to study the self-attracting polymer limit. The accuracy of our results is good enough to show the crossover of the tricritical behavior of the model as the limit is approached where it is equivalent to the SASAW model, but we were unable to distinguish between both values proposed in the literature for the tricritical exponent ν_t of the diluted-polymer model.

This paper is organized as follows. In Sec. II the model is defined. In section III, the methods we used to study the thermodynamic properties of the model on the square lattice are presented. We performed transfer-matrix calculations for the model defined on strips of finite width L and then used finite-size-scaling methods to extrapolate the results to the two-dimensional limit $L \rightarrow \infty$. Two different extrapolation methods were used: in Sec. III A PRG calculations are shown, and in Sec. III B the FOM is used. Finally, in Sec. IV, our results are discussed.

II. DEFINITION OF THE MODEL

In the model of equilibrium polymerization with annealed dilution each site of the lattice is occupied either by a monomer or by a solvent molecule. This may be described by associating an Ising lattice-gas variable μ_i to each lattice site i such that

$$\mu_i = \begin{cases} 1 & \text{if site } i \text{ is occupied by a monomer} \\ 0 & \text{if site } i \text{ is occupied by a solvent molecule .} \end{cases}$$

The fugacity of a solvent molecule will be considered equal to 1, and a monomer has fugacity z. First-neighbor interactions are considered between molecules on the lattice. Without loss of generality, we may set the Boltzmann factor of the solvent-solvent and the solvent-monomer interactions equal to 1, whereas the monomer-solvent interactions will have a factor y associated to them. The polymers will be modeled as SAW's constrained to pass through sites occupied by monomers. The activity of a polymer bond is x and usually an activity K_1 is associated to each chain. We will be interested in the limit $K_1 \rightarrow 0$, where a vanishing density of chains is present on the lattice and therefore in the thermodynamic limit only chains with an infinite number of monomers are considered. This model is equivalent to the n-vector model of magnetism with annealed dilution in the formal limit $n \rightarrow 0$ [3].

For calculations on one-dimensional strips of width L using transfer-matrix techniques, it is convenient to impose the existence of only one self-avoiding walk, whose endpoints are confined to the boundary [8,15–17]. This is equivalent to make the external field equal to zero in the magnetic analogy and break the symmetry on the boundary. Then, the (semi)grand-partition function for a chain on a lattice of N sites is

$$Y_N(x,y,z) = \sum_{n=0}^{\infty} \sum_{\{\mu_i\}} x^n y^{l(\{\mu_i\})} z^{m(\{\mu_i\})} \Gamma_N(n, \{\mu_i\}) , \qquad (1)$$

where n is the number of bonds in the chain, l is the num-

ber of first-neighbor monomer-monomer pairs, and m is the number of monomers on the lattice. For a given distribution $\{\mu_i\}$, l and m are given by

$$l(\{\mu_i\}) = \sum_{\langle i,j \rangle} \mu_i \mu_j, \quad m(\{\mu_i\}) = \sum_i \mu_i . \qquad (2)$$

The number of configurations on the lattice of N sites with n polymer bonds and distribution $\{\mu_i\}$ is given by $\Gamma_N(n, \{\mu_i\})$. The temperature of the model will be defined by

$$y = \exp(\beta), \quad \beta = \frac{1}{T} . \qquad (3)$$

The partition function for the interacting polymer model (SASAW) may be obtained from the partition function of the diluted model Eq. (1) in the limit $z \rightarrow 0$, $x \rightarrow \infty$, $xz = $const [9,11]. In this limit the only nonzero terms in the sum are those configurations where all the sites which are not incorporated into the chain are solvent molecules. With this constraint we obtain from Eq. (2), for the nonzero contributions, $l = n + k$, where n is the number of polymer bonds defined in Eq. (1) and k is the number of nearest-neighbor monomers which are not consecutive on the polymer. Thus, the partition function may be written as

$$Y_N(\alpha, y) = \sum_{\text{SAW}} \alpha^n y^k , \qquad (4)$$

where $\alpha = xyz$ and the sum is over all configurations of the SAW compatible with the boundary conditions. This is the partition function of a self-attracting polymer, and the tricritical point is this model is known as the Θ point [1].

Transfer-matrix methods for strips of finite width with periodic boundary conditions together with finite-size-scaling theory are used to study the locus of tricritical points of the diluted-polymer model on the square lattice.

III. TRANSFER-MATRIX CALCULATIONS

The finite-size-scaling theory [18,19] may be applied to obtain estimates of the critical behavior of models by studying their properties on finite lattices and then extrapolating the results to the thermodynamic limit. The method is quite powerful to study two-dimensional models, since it is often possible to calculate the transfer matrix for one-dimensional strips of finite width L, performing the limit of infinite length of the strip. Defining a pseudocritical temperature T_L for each strip, the finite-size-scaling properties of physical magnitudes (densities, correlation length, etc.) allow us to calculate approximate values for the two-dimensional critical temperature and critical exponents.

To build the transfer matrix for the diluted-polymer model, a straightforward generalization of the method used for the model without dilution [15] is used, including the lattice-gas monomer-solvent interaction. In this case, the nonpolymerized phase corresponds to an Ising lattice-gas system. Therefore the transfer matrix of this phase is not the 1×1 identity matrix as in the pure polymer case [15]. Therefore, the total transfer matrix is

block-diagonal, where one block is the symmetric transfer matrix of a Ising lattice gas, and the other block is the nonsymmetric transfer matrix of one single SAW in the presence of dilution.

There are several ways to choose a pseudocritical parameter for a model defined on strips of finite widths. A useful one, used in a great variety of two-dimensional systems, is the phenomenological renormalization group (PRG) [19,20], which has been applied to several polymer problems [15,16,21]. For pure- or diluted-polymer models, another natural way of choosing pseudocritical magnitudes exists, since in one dimension the diluted-polymer model exhibits a first-order phase transition [7,22], which is expected to occur for any finite strip. It is then possible to define the pseudocritical field variable T_L (the concept of field variable is used here in the general thermodynamic sense) as the exact first-order transition field of the finite system [8,16,17]. This was called the first-order transition method (FOM). We applied both methods to the diluted-polymer system on a square lattice. The boundary conditions for the strips were chosen to be periodic in the transverse direction and free in the longitudinal direction. The thermodynamic limit is taken letting the length of the strip grow to infinity with the width L fixed.

The number of possible configurations of the system, equal to the size of the transfer matrix, grows considerably as the width L of the strip is increased. The effective size of the matrix may be reduced using the symmetries of the model. In Table I a comparison of sizes of the transfer matrix for the nondiluted- and diluted-polymer problems with periodic boundary conditions, once the symmetries have been used, is shown. It is expected that the results of the calculations for the two-dimensional model are more and more accurate as the width of the strips considered is increased, since the extrapolations become more confident. The available computational resources set an upper limit to the widths which may be treated. Since the transfer matrix grows faster for the diluted-polymer model than for the nondiluted one, the target widths considered in the former problem are small-

er than the ones included in the latter one, and so is the accuracy of the results.

To obtain an extrapolated value of a quantity we use, when it is possible, the values obtained from the three largest L values with the expression [21]

$$\Phi_L = \Phi_\infty + \frac{A}{L^{\Delta_\Phi}} , \qquad (5)$$

where Φ may be the critical activity, a critical exponent, etc.; Φ_∞ is the extrapolated value; A and Δ_Φ are constants. For the diluted-polymer model, the few values obtained and the presence of parity effects forced us to restrict ourselves to graphical estimates.

A. Phenomenological renormalization group

In PRG calculations [19,20] we use the correlation length to define pseudocritical field parameters. The correlation length for the model on the strips is calculated using the transfer matrix through the expression

$$\xi_L = \frac{1}{\ln(\lambda_{np}/\lambda_p)} , \qquad (6)$$

where λ_{np} is the largest eigenvalue of the Ising lattice-gas matrix for the diluted-polymer model, and $\lambda_{np} = 1$ for the nondiluted model. In both cases λ_p is the largest eigenvalue of the polymer transfer matrix. For a critical point, the renormalization equation is [20]

$$\frac{1}{L} \xi_L(x) = \frac{1}{L'} \xi_{L'}(x') , \qquad (7)$$

and the estimate for the critical activity $x^{(L,L')}$ is a fixed point of Eq. (7). In order to optimize the method, it is convenient to choose $L' = L + 1$ [23]. However, for systems with parity problems (antiferromagnetic models [24], self-attracting polymers [16,17], polymers with annealed dilution [8], etc.) it is necessary to set $L' = L + 2$.

For estimating the location of tricritical points another equation is needed, and the PRG method for critical points may be generalized in different ways. One way is to consider three widths writing two equations of type Eq. (7) [25] for strips of widths L, L', and L''. Another one is to use the third largest eigenvalue of the complete transfer matrix [26]. For the diluted-polymer problem we found no acceptable solution for these equations. Something similar happens in the self-attracting polymer model [16], and therefore we used the same method employed there for obtaining estimates of the tricritical points and exponents. So, we calculate the value of $x^{(L)}(\beta,z)$ given by the PRG equation,

$$\frac{1}{L} \xi_L(x^{(L)},\beta,z) = \frac{1}{L+2} \xi_{L+2}(x^{(L)},\beta,z) , \qquad (8)$$

and estimate the exponent ν for each temperature and monomer activity as

$$\nu^{-1} = \frac{\ln[\xi'_{L+2}(x^{(L)},\beta,z)/\xi'_L(x^{(L)},\beta,z)]}{\ln(L+2/L)} - 1 , \qquad (9)$$

where ξ' denotes the derivative of ξ with respect to x. The use of widths L and $L+2$ is due to parity reasons.

TABLE I. A comparison of the sizes of the transfer matrices for pure- and diluted-polymer models for a strip of width L with periodic boundary conditions once the symmetries have been used.

L	Pure case	Diluted case
1	1	1
2	1	2
3	2	4
4	3	10
5	7	26
6	13	76
7	32	232
8	70	750
9	179	2 494
10	435	8 524
11	1142	29 624

Plotting ν against β at $z=z_0$ fixed, as in the cases of branched polymers [25] and self-interacting polymers [16], the curves corresponding to different values of L intersect almost at the same point, which is estimated as the tricritical temperature and the tricritical exponent ν_t.

This method also has problems for the diluted-polymer model. A study of the three largest eigenvalues of the complete transfer matrix over the tricritical line $z=z_{TC}(\beta)$ was made. We found that for the inverse temperatures smaller than $\beta^* \simeq 1.6$ (depending on the value of L) the second largest eigenvalue does not belong to the same block of the transfer matrix for the strips of widths L and $L+2$. The largest eigenvalue always belongs to the lattice-gas block, but for the second largest eigenvalue of the lattice-gas transfer matrix and the largest eigenvalue of the polymer transfer matrix we found that

$$\lambda_p^{(L+2)} > \lambda_{LG,2}^{(L+2)} ,$$
$$\lambda_{LG,2}^{(L)} > \lambda_p^{(L)} . \tag{10}$$

Because of this situation, the method was used only for the low-temperature region on the tricritical line, where the second largest eigenvalue is the largest eigenvalue of the polymer transfer matrix for the two sizes. We believe that this problem is due to the fact that above the Ising critical line $[\beta \geq 2 \ln(\sqrt{2}-1) \simeq 1.76, z=y^{-2}$ for the Ising lattice gas on the square lattice], the second largest eigenvalue of the lattice-gas transfer matrix asymptotically degenerates in the two-dimensional limit.

To study the correspondence between the diluted-polymer model and the SASAW model in the limit $z \to 0$, we calculate the critical exponent ν for small values of z. In Fig. 1(a) a plot of exponent ν against β for $z_0=10^{-6}$ and several values of L is shown. These values, in the high-temperature zone, are consistent with the exact value for SAW $\nu=3/4$. In the low-temperature zone, they tend to the collapsed value $\nu=1/2$. The curves in-
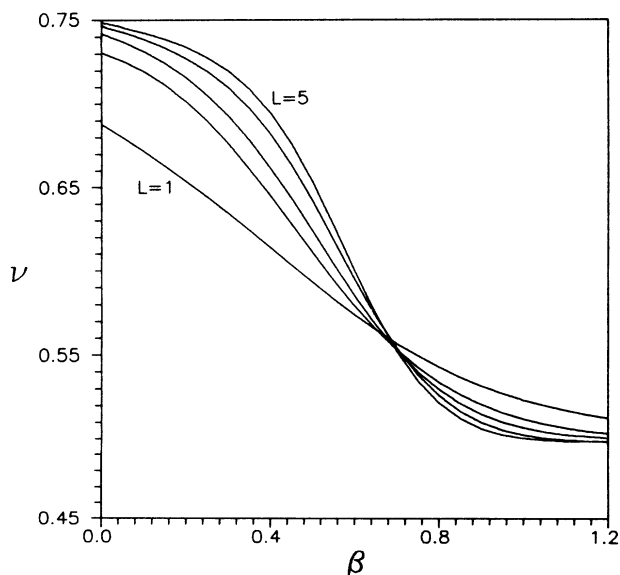
tersect almost at the same point which determines the estimates for the tricritical values of β and ν. The limit $z \to 0$ corresponds to the self-attracting problem, and we observe that this figure is graphically indistinguishable from Fig. 2 of Ref. [16] for the interacting polymer model. This confirms numerically the relation between both models in this limit.

In Fig. 2 the tricritical exponent ν_t against β is plotted. There, it is observed that the values for ν_t are compatible with the exact Θ-point tricritical exponent $\nu_t=4/7$ [27] for small values of β, which corresponds with the $z \to 0$ limit, and increase rapidly with β, stabilizing at a value compatible with the conjectured tricritical exponents $\nu_t=8/11$ [13] and $\nu_t=3/4$ [14] for the diluted-polymer model. This result indicates a crossover in the tricritical behavior of the diluted-polymer system, as the limit where the model is equivalent the SASAW model is approached. As was stated above, due to compositional restrictions the largest width we considered for the strips was $L=7$. So, the curve labeled by 3 in Fig. 2 represents the estimates for ν_t obtained from the intersection of the estimates for ν obtained from strips of widths $L=3$, $L'=5$, and $L''=7$. It is clear in the figure that the accuracy of our calculations is insufficient to distinguish between both conjectured values for ν_t, but, on the other side, the crossover to the Θ-point behavior is clearly visible. Also, the widths we were able to consider are not high enough to allow an extrapolation of the estimates for ν_t through Eq. (5).

B. First-order phase-transition method

In this case, the pseudocritical parameter is calculated as the exact first-order critical parameter for a finite strip, that is, $x^{(L)}(\beta,z)$ is given by the condition
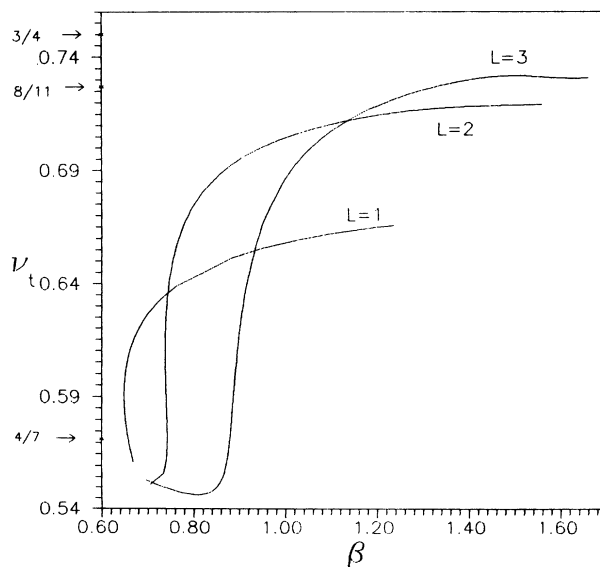

FIG. 1. Curves of ν vs β for $z=10^{-6}$ and $L=1,\ldots,5$. For comparison with SASAW see Fig. 2 of Ref. [16].

FIG. 2. Estimates for the tricritical exponent ν_t against β for $L=1,2,3$ using the PRG. The values conjectured to be exact for the Θ point (4/7) and for nonzero values of z (8/11 or 3/4) are indicated by arrows.

$$\lambda_p^{(L)}(x^{(L)},\beta,z)=\lambda_{LG}^{(L)}(\beta,z) , \tag{11}$$

where $\lambda_p^{(L)}$ and $\lambda_{LG}^{(L)}$ are the largest eigenvalues of the polymer block of the transfer matrix and the lattice-gas block, respectively. Unlike for the PRG calculations, we had no numerical problems using the FOM at any temperature.

The polymer density ρ of the model defined on a strip of finite width L may be calculated using Eq. (1):

$$\rho_L(x,y,z)=\lim_{N\to\infty}\frac{x}{N}\frac{\partial}{\partial x}\ln[Y_N(x,y,z)] . \tag{12}$$

In the transfer-matrix formulation, it is given by

$$\rho_L(x,y,z)=\begin{cases}0, & x<x^{(L)}\\ \dfrac{x}{L\lambda_L}\dfrac{\partial\lambda_L}{\partial x}, & x>x^{(L)} .\end{cases} \tag{13}$$

To allow a comparison with phenomenological renormalization, this method was applied to the nondiluted case ($z\to\infty$). In this case, the value of the polymer activity at the first-order phase-transition point is given by $\lambda_L(x^{(L)})=1$, where λ_L is now the largest eigenvalue of the polymer block. In Fig. 3, the polymer density $\rho_L(x)$ is plotted against x. At the first-order phase-transition activity $x^{(L)}$, the polymer density is not continuous, and the discontinuity in the density decreases with L, leading to the extrapolated value $\rho_\infty=-0.005$, consistent with the two-dimensional value $\rho_c=0$ for a continuous phase transition. The extrapolating curve for the critical polymer density, calculated using Eq. (5) with the three largest values of L (11, 10, and 9), is also plotted.

The estimates for the critical exponent ν are calculated from the finite-size-scaling form of the polymer density [25]:

$$\rho_L(x)\sim L^{1/\nu-2}F((x-x^{(L)})L^{1/\nu}) . \tag{14}$$

Then, since in this case no parity effects were observed, estimates for ν may be obtained using Eq. (14) with sizes L and $L+1$:

$$\nu^{-1}=\frac{\ln[\rho_{L+1}(x^{(L+1)})/\rho_L(x^{(L)})]}{\ln(L+1/L)}+2 . \tag{15}$$

The extrapolation to $L\to\infty$ gives $x_c=0.3797$, $\nu=0.7498$. These values are very close to the corresponding values obtained using phenomenological renormalization in Ref. [15] $x_c=0.37905$, $\nu=0.7503$.

The diluted-polymer system can be studied by this method using Eqs. (11) and (13) to obtain the critical polymer activity $x^{(L)}(\beta,z)$ and the critical density $\rho_L(x^{(L)},\beta,z)$. In Fig. 4, plots of the critical density as a function of temperature for two fixed values of the monomer fugacity are shown. In both cases we notice that for $\beta\to0$ and for high β the critical density value is the one obtained for the nondiluted model (characterizing a con-
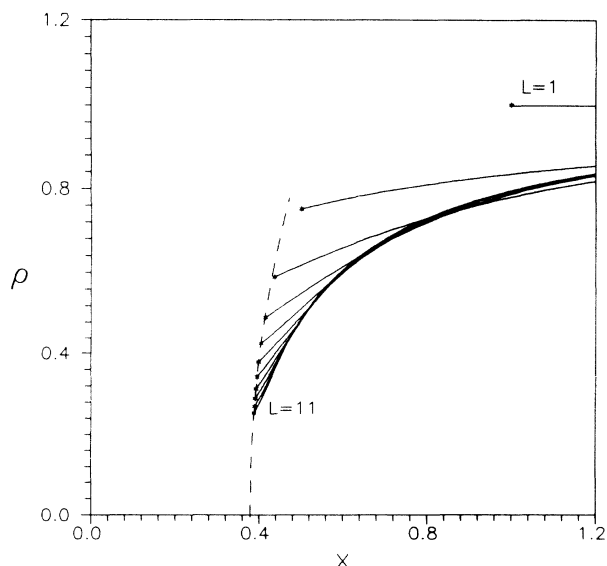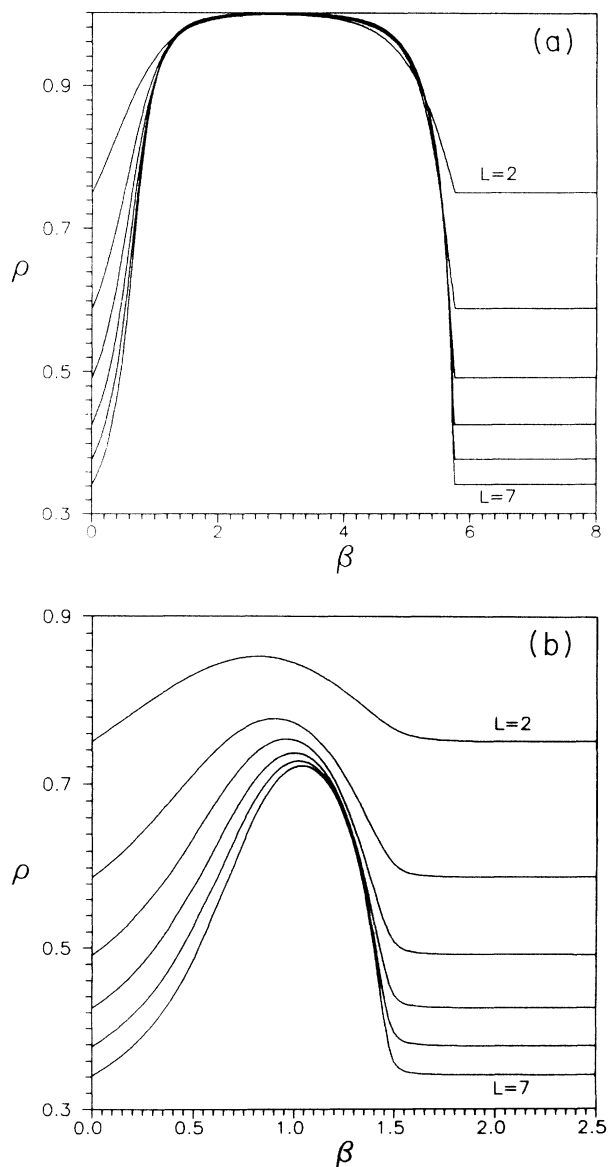

FIG. 3. The polymer density ρ_L vs x for the pure polymer case for $L=1,\ldots,11$. The dashed line corresponds to the extrapolating line for the critical density.

FIG. 4. ρ_L vs β for $z=z_0$ fixed. (a) $z_0=10^{-5}$, (b) $z_0=0.0525$.

tinuous phase transition in the limit $L \to \infty$). However, it increases rapidly for intermediate values of β, giving the idea of a first-order phase-transition zone, limited by two tricritical points. For high values of z, we found that the critical density is a constant (equal to the nondiluted value) as a function of β.

To estimate the tricritical line and the tricritical exponent ν_t we used the finite-size scaling form of the critical density near a tricritical point [16]:

$$\rho_L(T) \sim L^{1/\nu_t - 2} \widetilde{F}((T - T_{TC})L^{\phi_t/\nu_t}) . \tag{16}$$

Following the arguments of Derrida and Saleur [16], we studied the behavior of the quantity

$$[X_L(\beta,z)]^{-1} = \frac{\ln[\rho_{L+2}(x^{(L+2)},\beta,z)/\rho_L(x^{(L)},\beta,z)]}{\ln(L+2/L)} + 2 . \tag{17}$$

As a consequence of Eq. (16), at a tricritical point X_L should be independent of L. To obtain the tricritical line, we fixed $z = z_0$ and the intersection of the curves $X_L(\beta,z_0)$ for two consecutive values of L gives an estimate for the tricritical temperature $\beta_L(z_0)$ and for the tricritical exponent

$$\nu_t^{(L)}(\beta_L,z_0) = X_L(\beta_L,z_0) .$$

The correspondence between the diluted-polymer model and the self-interacting polymer model in the $z \to 0$ limit may be appreciated in Fig. 5, where X_L is plotted as a function of β for several values of L and for $z = z_0 = 10^{-6}$. The curves are graphically indistinguishable from those Fig. 1 of Ref. [16] for the self-attracting chain confirming numerically, as in the phenomenological renormalization calculations, this equivalence. It should be noted that the curve for $L = 6$ is not present in Fig. 1 of Ref. [16]. The value obtained for the tricritical parameters are listed in Table II. In Refs. [16,17], the es-
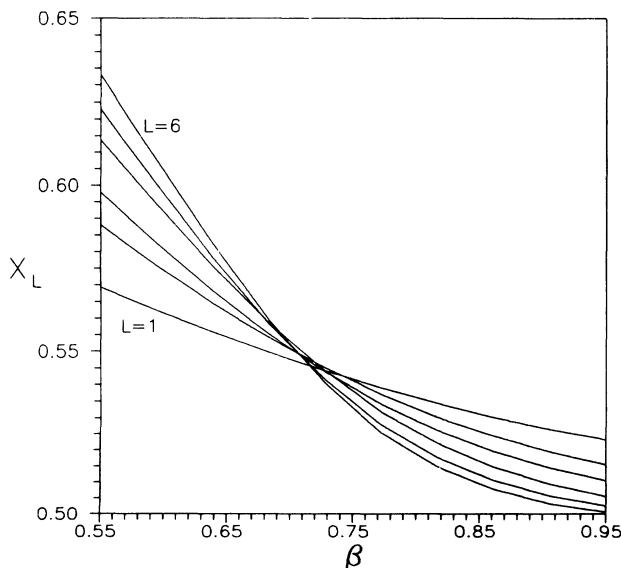
TABLE II. Estimates for the tricritical temperature and the tricritical exponent ν_t for the Θ point for the SASAW obtained with widths $L, L+2, L+4$, using the first-order transition method. The estimates are defined as the intersection points in the curves in Fig. 5.

L	β_Θ	ν_t
1	0.724 848	0.544 424
2	0.718 547	0.546 711
3	0.706 250	0.549 030
4	0.689 752	0.556 274

timated value $\nu_t = 0.55$ is notably smaller than the conjecture for the exact value $\nu_t = 4/7 \simeq 0.57$. The value for $L = 4$, $\nu_t^{(L=4)} \simeq 0.56$ obtained in our calculations shows that the estimate of Refs. [16,17] is too small, and our results are in better agreement with the result supposed to be exact [13].
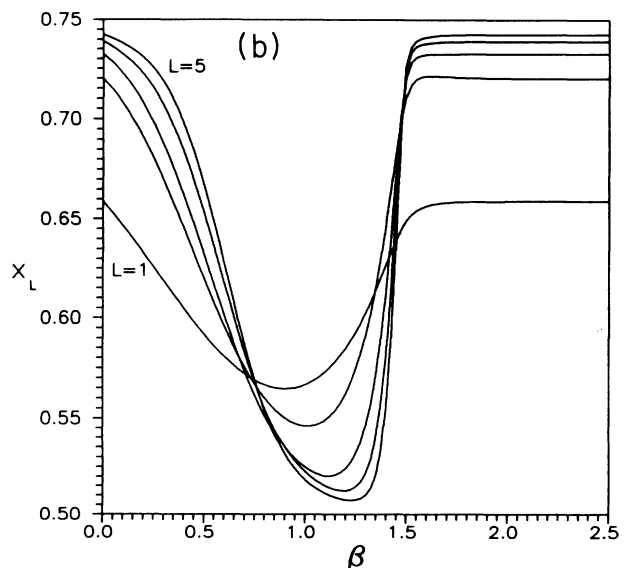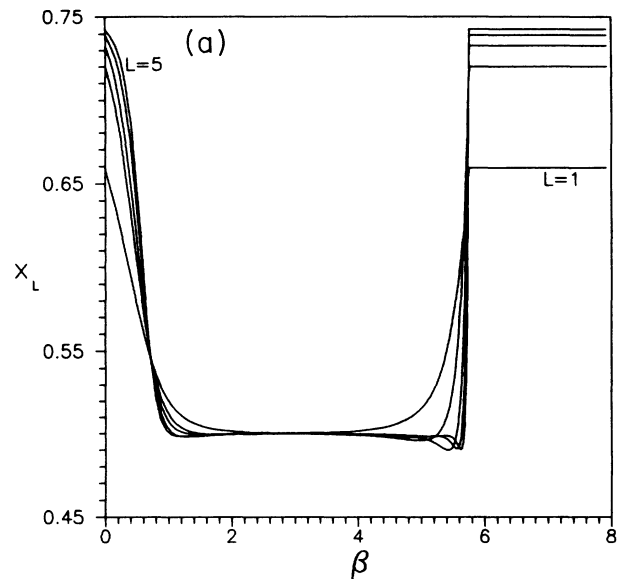

FIG. 5. X_L vs β for $z = 10^{-6}$ and $L = 1, \ldots, 6$. For comparison with self-attracting polymers, see Fig. 1 of Ref. [16].

FIG. 6. X_L vs β for $z = z_0$ fixed with the same values for z_0 of Fig. 4. (a) $z_0 = 10^{-5}$, (b) $z_0 = 0.0525$.

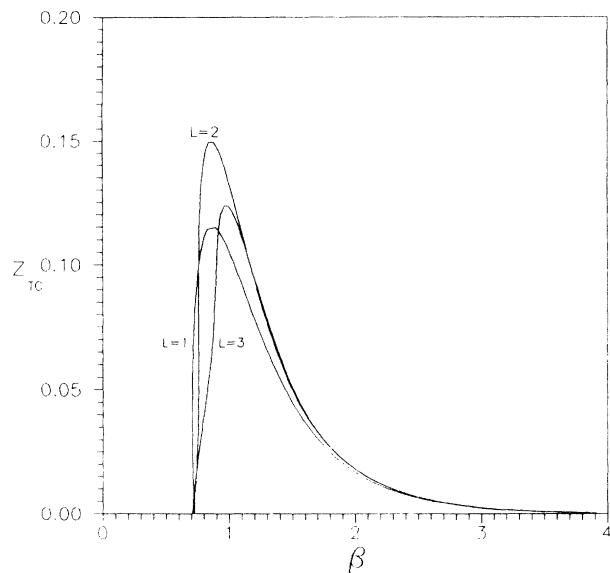FIG. 7. z_{TC} vs β. The curves labeled by a number L are obtained from FOM for strips of width $L, L + 2, L + 4$.
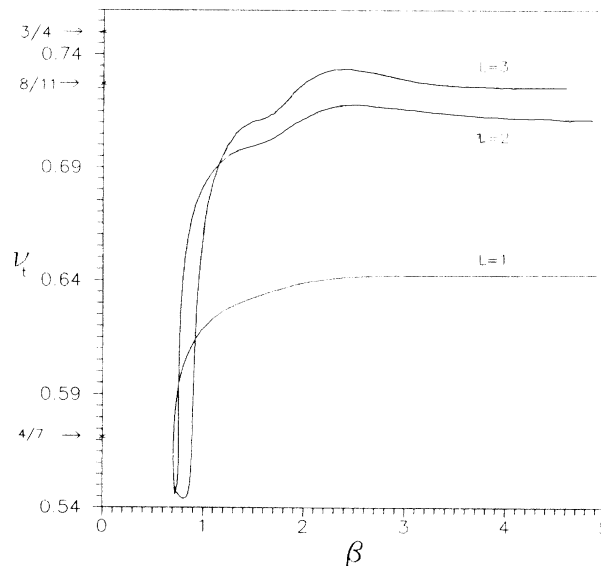

FIG. 8. Estimates for the tricritical exponent ν_t against β using FOM. As in Fig. 2, the values conjectured to be exact are indicated by arrows.

In Fig. 6 we plot X_L vs β at $z = z_0$ fixed, for the same values of z_0 as in Fig. 4. From the intersection of these curves, it is clear that two tricritical points with different values of the exponent ν_t are present.

For each value of z_0 we obtain two values of the tricritical inverse temperature β_L and for the tricritical exponent ν_t^L. With those calculations the tricritical curves z_{TC} vs β shown in Fig. 7 were obtained. Estimates of the tricritical exponent ν_t were also obtained and they are shown in Fig. 8.

These results support the same conclusions of Sec. III A that a crossover occurs in the tricritical behavior of the diluted-polymer model, as the limit in which the model is equivalent to the SASAW model is approached.

IV. CONCLUSIONS

The model of polymers with annealed dilution is studied using transfer-matrix methods and finite-size-scaling arguments. Two different methods were used to define the pseudocritical parameters for finite strips. Both methods were compared to similar calculations in the literature in the pure-polymer limit, and numerically identical results were obtained.

Our results indicate that a crossover occurs in the tricritical behavior of the diluted-polymer system in the limit $z \to 0$, $x \to \infty$, $xz = $const. Figures 2 and 8, obtained by PRG and FOM, respectively, show that the value of the tricritical exponent ν_t is compatible with the tricritical exponent of the Θ point $\nu_t = 4/7$ for $\beta = \beta_\Theta$, which corresponds to the SASAW limit, and a considerably higher value of ν_t is obtained for $\beta > \beta_\Theta$, or equivalently, for nonzero values of z. The accuracy of our estimates for ν_t of the diluted-polymer model is not good enough to distinguish between both conjectured values present in the literature, which are $\nu_t = 8/11$ [12,13] and $\nu_t = 3/4$ [14], identical to the critical value of ν for pure SAW's.

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